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AN APPROACH FOR THE TOPOLOGY DESIGN AND OPTIMIZATION OF A UTILITY NETWORK WITH UNRELIABLE ELEMENTS

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Abstract. We consider a problem of the topology design and optimization of a utility communication network. Mathematically, it is represented by a hierarchical two-level network which allows us to take into account the fact that a communication line (trench or tunnel) of a primary network can be used multiple times for laying edges (various utility communications) of a secondary network while the expenses for preparation of that line are made just once. Another advantage is that we can consider a failure in a primary network along with its influence on all secondary network elements lying in it. For a given topology of a secondary network and a redundant topology of a primary network, we analyze the complexity of the problem of the cheapest choice of the primary network elements for laying into them secondary network elements. The methods for solving this problem are proposed, along with results of the numerical experiments.

Keywords: graph, hypernet, utility network, network reliability, network optimization, heuristic algorithms.

AMS Subject Classification: 05C90; 05C85; 93A15; 93A30; 68Q25

1. Introduction

Modern utility networks serve industrial facilities, mining companies, living areas, shopping malls, cultural and household organizations, and other infrastructure objects. All this is a complex system consisting of gas/oil pipelines, water pipelines, heating networks, power supply networks, communication networks, monitoring networks, etc. Such networks can be underground, ground, or aerial.

A fundamental feature of the utility communication system is its hierarchical nature. As a rule, such systems can be described mathematically as two-level networks. For example, lines of communication for various purposes can be located in one trench. Thus, there is a network of trenches (a network of the first level), and a set of various networks laid down in it (secondary networks).

Let us consider, for example, networks for providing the work in mines, including a power supply network, a monitoring network, a network for providing wireless communication for personnel. Physically, all these networks are located in a network of mine tunnels, which in this case is the primary network containing many secondary ones. The need to consider it in the framework of the same mathematical model is due to the fact that a collapse of a mine tunnel can lead to a break in all communications passing through it. To simulate such objects, it is convenient to use the hypernet apparatus [18]. In the general case, we can combine all the secondary networks into one network, which is represented by a multigraph with the use of markers for describing different secondary networks. Further, we consider the above described case of the element failure, i.e. elements of the primary network are subject to random failures.

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It is known that in modern cities a system of utility networks is a complex branch of urban economy, the share of the cost of facilities and objects of which exceeds 30% [22]. In this regard, the task of a cost reducing to building and operating utility networks for various purposes becomes more and more actual. On the other hand, along with economic criteria, various technical criteria must be considered, such as reliability, compatibility of various networks, etc. The compatibility is one of important factors when various utility networks are laid in the same technical corridor. So we need to take into account the compatibility or incompatibility of these networks and a normative distance between them. Considering the hierarchical nature of networks allows all these factors to be taken into account.

In the general case, we arrive at the problem of designing a system of utility networks for various purposes (a primary network and secondary networks), which has a minimum cost and providing a sufficient reliability and compatibility of various secondary networks. We offer to formulate and solve such problems as the problem of mapping the secondary network into the primary network of some hypernet HN [18] with given restrictions. This allows us to take into account the redundancy of the primary network. That is, in the case of designing not only the secondary network, but also the primary network along the way, it is assumed that a certain subgraph should be formed from the graph of the primary network, containing all the elements of the secondary network, which describes the physical network.

The hypernet problems usually have a high complexity and belong to the class of NP-hard problems [17] which makes it difficult to solve them by exact methods for an appropriate large scale. In this paper, we analyze the complexity of problems of utility networks design in terms of hypernets and study the models and metaheuristic methods for solving such problems.

The rest of the paper is organized as follows: in Section 2, mathematical models and methods for the utility networks design are presented. In Section 3, we formulate the problems to be solved, whose belonging to the class of NP-hard problems we prove in Section 4. Sections 5 and 6 describe the methods we propose and the results of numerical experiments. Section 7 is a brief conclusion.

2. Mathematical models and methods for the utility network topology design

Currently, there is a number of approaches for laying the utility networks in a given area. Most of them have an assumption that a network is a two-dimensional object placed on a given surface.

In [1, 9], models and methods for reducing construction and operation cost of backbone networks and distribution networks are proposed. In [4–6], the authors study the problem of the gas transmission network topology design in order to minimize the construction cost and solve it by linear and nonlinear programming methods and the simulated annealing algorithm. The design of the water supply network based on simulation and probabilistic analysis models, as well as on the hydraulic networks model, is ‘ studied in [12, 23, 26]. Redundant schemes [14], alternative trees and graphs [24], and tensors and morphological tables [11, 15] are used as various methods for finding an optimal topology during the network design.

To describe the utility networks topology we can use graphs or hypergraphs [3], sandwich graphs [7], nested graphs [19], multilayer networks [2, 16], multilevel complex networks [10].

Here we treat the problem of the utility network design with the use of a two-level hypernet, defined as follows:

Definition: Hypernet $HN = (X, E, R, F)$ is a hierarchical mathematical model for representing the utility networks topology:

- at the lower level:
  $X = (x_1, x_2, ..., x_n)$ is a set of vertices;
  $PN = (X, E)$ is a graph corresponding to the discrete analogue of the physical area for

the network location. We call it *primary network*, where $E = (e_1, e_2, ..., e_g)$ is a set of edges of PN graph, which we call *branches*.

- at the upper level: $SN = (Y \subseteq X, R)$ is a graph corresponding to the designed network, the so-called the *secondary network*, where $R = (r_1, r_2, ..., r_m)$ are the edges of the graph which are routes in PN graph;
- $F : R \to 2^V$ is a one-to-one mapping between the edges $r \in R$ of the secondary network graph SN and the routes in the primary network PN formed by the branches $e \in E$.

Note, that PN and SN are assumed to be undirected graphs.

For a branch $e \in E$ of the primary network PN, we introduce the following notation:
$a(e)$ is the cost of construction of the branch $e \in E$. It consists of all expenses for e construction, such as excavation, materials costs, and other.
$l(e)$ is the geometrical length of the branch $e \in E$.
Also, for problems of the hypernet reliability analysis [13], we assume that the values $p(e)$ are given, that are the probabilities of existence of branches. Further, we call these values *reliabilities* of branches.

For an edge $r \in R$ of the secondary network graph SN:
$l(r) = \sum_{e \in F(r)} l(e)$ is the geometrical length of the edge $r \in R$.
$b(r) = b'(r) \cdot l(r)$ is the cost of construction of the edge r. It is determined as product of the edge length by the unit cost of construction per unit length $b'(r)$.
$p(r) = \prod_{\forall e \in F(r)} p(e)$. So, *reliability* of an edge we define as product of reliabilities of all branches contained in this edge. As reliability of the whole hypernet we consider a minimum among all the values of each edge reliability:

$$R(HN) = \min_{r \in R} p(r).$$

The analysis of the reliability of hypernets was previously studied in [20, 21].

In Fig.1, the primary network PN is represented as a 3×3 grid, and the secondary network graph is represented as a set of routes embedded in PN: $R = \{(1, 4, 7), (1, 4, 5), (7, 8, 9)\}$.

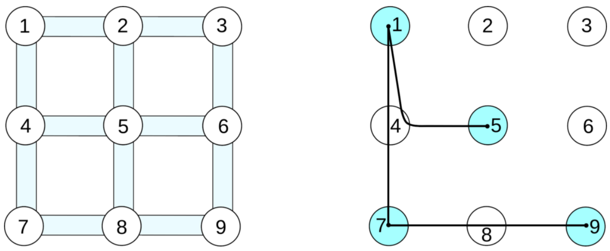


FIGURE 1. An example of the two-level hypernet HN

We can see, that the secondary network SN implemented into the primary network graph PN has a tree topogy. In the general case, the secondary network graph SN can be a line graph, a tree, a complete graph, a partial bipartite graph, etc.

Note that in the framework of this model we consider a utility communication system which is located underground. So, if there already a trench exists for a communication channel of a secondary network, we can place in it another communication channel (of the same or another secondary network) without excavation or preparation expenses. In some cases, this rule can hold for an utility communication system which is located on the ground or aerial. If so, the following methods can be applied for the design of such utility communication systems.

As a secondary network, we consider a graph obtained by the union of all secondary utility networks. If there is any multiple edge in this union, we transfer it into one edge r modifying the value $b(r)$. In the general case, S-hypernet [18] can be used to simulate a group of secondary networks. However, for describing the problems under consideration, we choose the hypernet with combined secondary networks, since S-hypernet apparatus is disproportionally complicated.

3. Statement of the networks optimization problems

In this section, we introduce formulations of problems of the network topology optimization in terms of the hypernets theory.

Problem 1. Let us assume that topologies of the graphs PN and SN are given. The problem is to find the hypernet HN of a minimum cost, i.e. the hypernet with mapping F such that the following function takes a minimum value:

$$Q(HN) = \left(\sum_{e \in F(r), \forall r \in R} a(e) + \sum_{r \in R} b(r) \right) \to min \tag{1}$$

The network optimization problem in the form of (1) is the most general in the two-level hypernet case and has many applications for describing and optimizing topology of networks for various purposes, such as laying pipelines. As is described above, the main point is that a branch of a primary network can be used multiple times for laying edges of a secondary network while the expenses for branch preparation are made once.

Problem 2 is a modification of Problem 1 taking into account the possibility of failures of primary network elements. Let we have given topologies of the graphs PN and SN and the values $p(e)$ of the existence probability of primary network branches. Also, we have a predefined value $0 < P_0 \leq 1$, which is a lower reliability threshold of the hypernet sought. The problem is to find the hypernet HN of a minimum cost, i.e. the hypernet with mapping F such that function 1 takes a minimum value and the following inequality holds:

$$R(HN) \geq P_0.$$

Other two problem, we studied earlier, are problem 1 with the assumption that we have a number of additional Steiner points for location and problem 1 with the assumption that we have a number of secondary networks of different types which can be compatible or can be not for laying in the same branch or not. Here we do not provide their formulation, it can be found in [**?**] along with the methods for their solving and results of numerical experiments.

4. The analysis of complexity of the problems stated

Let us find out whether the problems stated are solvable within polynomial complexity. For Problem 1, it is so assuming the certain restrictions:

Lemma 1. *Let costs of the primary network branches are null: $a(e) = 0$, $\forall e \in E$. Then the complexity of Problem 1 is polynomial.*

Proof. Let us assume that the PN branches costs are negligible in comparison to the SN edges costs. So, we may assign $a(e) = 0$ $\forall e \in E$. In this case the hypernet cost depends only on the second term in expression (1), i.e. $Q(HN) = \sum_{r \in R} b(r)$. Since $b(r)$ are linearly dependent on the length of r, we have to find all the shortest paths between the pairs y_i, y_j, $i \neq j$. The solution of such a problem can be found by Floyd algorithm within the polynomial complexity. □

However, assuming that branches costs are equal to zero, we obtain NP-hard problem even in this simplified form:

Lemma 2. *Let the costs of the secondary network edges be null: $b(r) = 0$, $\forall r \in R$. Then Problem 1 is NP-hard.*

Proof. Now we consider the case when the secondary network edges costs are negligible in comparison to the primary network branches costs. Thus, the hypernet cost depends only on the first term in expression (1), i.e. $Q(HN) = \sum_{e \in F(r), \forall r \in R} a(e)$.

Let us assume that $|R| \geq 2$, otherwise *Problem 1* can be solved by the Dijkstra algorithm within polynomial complexity.

(1) Let us consider the case when SN is connected. Below we show that *Problem 1* solution is a minimal cost subtree of the primary network PN. Otherwise, the embedding SN in PN is not a subtree in the primary network PN, i.e. the minimum cost solution has cycles either in the primary network PN or in the secondary network SN. We will show that this leads to a contradiction. Let us consider two cases regarding the topology of SN:

(1.1) SN is a tree.

a) Let some two routes AD and AC of the solution form a cycle (Fig. 2a), where B is the intersection of these routes. We introduce the following notation: m_1 and m_2 are the routes parts AD and AC to node B from A, respectively, and $|m_1|$ and $|m_2|$ are their cost.
If $|m1| > |m2|$, then the route m_1 can be implemented along m_2. In this case, the hypernet cost decreases by $|m_1|$, and we arrive at a contradiction since there is a more cheap solution.

b) Let there be three routes AD, AE, EC forming a cycle (Fig.2b), where B is the intersection of AD and EC, m_1, m_2, m_3 are parts of the routes AD, AE, EC to the node B, respectively, $|m_1|$, $|m_2|$, $|m_3|$ are the cost of these parts of the routes.
If the cost of m_1 is not less than the total cost of m_2, m_3, i.e. $|m_1| \geq |m_2|+|m_3|$, then m_1 can be laid along m_2 and m_3 (AEB). Then the hypernet cost can be decreased by $|m_1|$, and we arrive at a contradiction since there is a cheaper solution.
If the cost m_1 is less than the total cost m_2 and m_3, and the cost m_2 is less than the cost m_3, i.e. $|m_1| < |m_2|+|m_3|$ and $|m_2| < |m_3|$, then m_3 can be laid along m_2 and m_1 (EAB) (Fig.2c). Then the hypernet cost can be decreased by $|m_3|$, and we arrive at a contradiction since we obtain a cheaper solution.

c) Let now there 4 or more routes form a cycle. In fact, this case is similar to the previous one. As shown above, the most expensive route can be re-laid along the rest of the routes, which decreases the hypernet cost and leads to a contradiction since we obtain a cheaper solution.

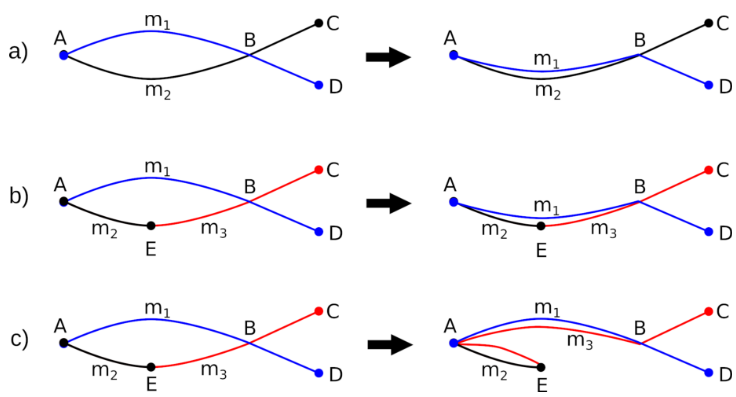


FIGURE 2. Assumption that routes in the primary network PN form a cycle

Thus, if the structure of secondary network SN is a tree, then the optimal primary network PN routes also form a subtree and a minimum cost hypernet corresponds to a subtree of the primary network PN.

(1.2) Let now the topology of secondary network SN be not a tree. Below we show that a minimum cost hypernet forms a subtree on the primary network PN, too.
Let us assume the opposite, i.e. some optimal routes in a primary network form cycles. Let now there three routes AB, AC, CB in the graph, which form a cycle (Fig.3). Among the routes AB, AC and CB, we choose the highest cost route (for example, m_1) and lay it along the routes m_2 and m_3 (ACB). Then the hypernet cost is decreased by $|m_1|$ and we arrive at a contradiction since we obtain a cheaper solution. The other cases of an interposition of routes in a cycle are considered exactly as is done above for the tree SN topology.

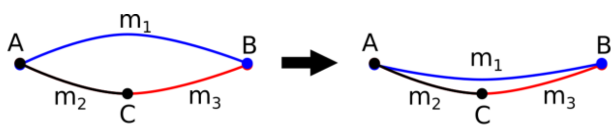


FIGURE 3. Assumption that routes in the primary network PN have a cycle

(2) Now, let the graph SN be not connected (Fig.4a), i.e. the problem solution is the forest.
We introduce a fictitious vertex v' (Fig.4b) and connect it by fictitious edges and zero-cost branches to each connectivity component of SN. Note that each connectivity component is connected by one and only one edge.
Then, the graph SN with a defined additional vertex v' and fictitious edges and branches is connected. Obviously, *Problem 1* solution is a minimum cost subtree of the primary network.

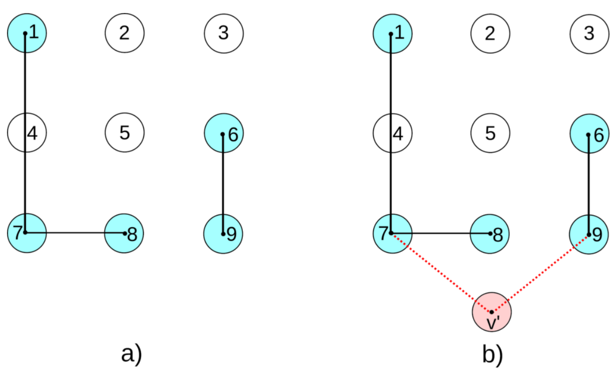


FIGURE 4. a) The disconnected graph SN; b) The connected graph SN

The minimum cost subtree of a graph is a Steiner tree in a graph. Therefore, we have shown that the Steiner tree is a solution of Problem 1 in the simplified form. The nodes that need to be connected by this tree (let us call them terminals) are the nodes that are incident to any edge of the secondary network.

Obviously, the opposite is also true, i.e. for solving the problem of finding a Steiner tree in a graph, we can solve problem 1 in the above simplified form. For this purpose, we specify the graph of the secondary network so that its nodes are terminals, and each terminal is incident to a branch.

Therefore, we have proved that Problem 1 in a simplified form is equivalent to the problem of finding a Steiner tree in a graph. It is known that the problem of constructing a Steiner tree in a graph is NP-hard [8]. Thus, Problem 1 is NP-hard, too. □

5. Methods for the secondary network laying

In this section, we consider using various techniques for an approximate solution to *Problem 2*, which is obviosly NP-hard since it includes NP-hard *Problem 1.* Preliminarily, we must find out whether the problem is solvable. For this purpose, let us design a maximum reliable hypernet.

By the definition, we have $p(r) = \prod_{\forall e \in F(r)} p(e)$. Let us consider a new graph of the primary network PN', in which the length of a branch $e \in E$ equals $-\ln p(e)$ instead of $l(e)$. According to the logarithm property $\ln p(r) = \ln(\prod_{\forall e \in F(r)} p(e)) = \sum_{\forall e \in F(r)} \ln p(e)$.

Next, we can use the Floyd Algorithm for PN' to find the laying routes for all edges. So, the obtained shortest path between the pair (y_i, y_j) in PN' is a most shortest in terms of $\sum -\ln p(e) = -\sum \ln p(e)$.

Since $0 \leq p(e) \leq 1$ for each e, $\ln p(e) \leq 0$. Therefore, to minimize $-\sum \ln p(e)$ is equivalent to maximization of the negative expression $\sum \ln p(e)$. According to the above mentioned property of the logarithm and its monotonicity, such optimization is equivalent to maximization of $\prod p(e)$ and the obtained shortest path between the pair (y_i, y_j) in PN' is a most reliable path.

Then, for an edge $r \in R$ we take a corresponding pair (y_i, y_j) and obtain using this approach a path for laying the edge in PN with the highest reliability. The route no-failure operation probability is calculated as $p(r) = \prod_{\forall e \in F(r)} p(e)$. If the obtained values satisfy the given threshold, then the problem is solvable. We call this algorithm *MaxProb* algorithm.

This algorithm takes into account only the probabilities, not the branches cost. Therefore, the solution satisfies the reliability constraint, but can be very expensive. Nevertheless, it can be used as a starting point for approximate algorithms to be discussed further.

For example, the *Greedy* approach [25] can be adapted taking into account the branches cost and reliability. The algorithm below searches for minimal cost paths satisfying the reliability condition.

5.1. **The FloydGreedyProb algorithm.**

Step 1. For each $r \in R$ do

Step 2. Find all shortest paths in PN between the endpoints (y_i, y_j) of r using the Floyd algorithm.

Step 3. Choose among them the path that satisfies the reliability constraint. Lay the edge $r \in R$ along it. If there is no such path, take the path found by the *MaxProb* algorithm.

Step 4. Assign $a(e) = 0$ for all $e \in F(r)$ in PN since the branch cost is zero for laying the remaining edges.

The biology-inspired techniques can also be used to solve Problem 2. For example, ant colony approach. The *AntColony* algorithm, based on this approach, is provided in our previous paper [25] Note we assume that if the "ants" could not find a path for an edge, then we find a path by the *MaxProb* algorithm. But, as is noted above, the *MaxProb* algorithm does not consider the branches cost.

Let us now consider the k-shortest path method for solving the problems stated. This well-known method is based on ordering a set of available alternative routes in a graph for [25]. The Yen algorithm [3] can be used for obtaining this set.

Below we describe a modification of the k-shortest path algorithm, which allows us to find lesser cost paths providing the reliability requirement.

5.2. **K-shortest paths algorithm (k-path).**

Step 1. Find the paths between the given pairs vertices (y_i, y_j) using the *MaxProb* algorithm.
Step 2. Order edges $r \in R$ of the secondary network SN in ascending order of their weight.
Step 3. For each edge $r_i \in R$ do
Step 3.1. Find all k-shortest paths as an ordered list of available alternative routes in the graph. For this purpose, we use a modification of the Yen algorithm:
Step 3.2. Sort all branches contained in it by the cost descending. Let us denote the first branch by e.
Step 3.3. Delete the branch e. For the edge r_i, find the shortest one by the cost path in the new primary network graph with the use of the Dijkstra algorithm.
Step 3.4. If a new path satisfies the reliability constraint and has a lesser cost than the current solution, save it as a new current solution.
Step 3.5. Return the branch e to the primary network. Denote by e the next branch if it exists, according to the ordering at Step 3.2.
Step 3.5. Return to Step 3.3.

This algorithm can be used as part of other algorithms:

- **Greedy+ k-path** is the *FloydGreedyProb* algorithm which uses the paths found by the *k-path* algorithm at Step 3.1.
- **AntColony+ k-path** is the *AntColony* algorithm, in which for paths that are not found, the paths found by the *k-path* algorithm are used.

Below, we numerically investigate the stated algorithms to compare which is better depending on input data.

6. Numerical experiments

In Fig.5 we present the comparison of the above described algorithms for solving Problem 1. As input data we have chosen the following:

The primary network PN is 10×10 grid. For each $e \in E$:
the branch cost $a(e)$ is a random integer number from 5 to 10 conventional units;
the length $l(e)$ is a random number from 1 to 5;
the relaibility $p(e)$ is a random real number from 0.9 to 0.99;
$b'(e) = 1$.

The secondary network is defined by pairs of vertices that are randomly chosen from PN nodes.

As k value we take 10.

As reliability constraints the following values are defined:
$P_0 = 0.7$ for $|R| = 5, 10$,
$P_0 = 0.6$ for $|R| = 30, 60$,
$P_0 = 0.5$ for $|R| = 80$.

For a large $|R|$, a smaller threshold was taken due to the fact that otherwise the problem was often unsolvable. If we take a small threshold on a small $|R|$, then the reliability constraint has almost no effect on the solution, since almost all found shortest paths satisfY the reliability constraint.

In Fig.5, the ordinate shows the hypernet cost, the abscissa shows the edges number $|R|$ that must be nested in the primary network PN.

We can see that the combination of *Ant Colony + k-path* algorithms finds a lower cost solution than just *AntColony*.

For small $|R|$ values, the best solution is found by *AntColony* family algorithms, and for large values, greedy algorithms (*Greedy*) work better.

It is interesting to note that the *k-path* algorithm finds a lower cost solution than *MaxProb* only for small values of $|R|$.

Although the *k-path* algorithm finds each individual path which is no more expensive than *MaxProb*, it does not consider that branches cost are considered only once, no matter how many edges have passed through it.

As a result of the *MaxProb* algorithm we have that the more reliable is a branch, the more possible paths pass through it, and in the *k-path* algorithm, looking for a lower cost solution, the edges are grouped worse in some branches, hence the final solution is more expensive. But when the *k-path* algorithm is used as another algorithm part, it can find a lower cost solution. In total, the best methods for Problem 2 solving are the methods based on k-path approach: *AntColony+k-path* or *Greedy+k*-path, depending on the input data.

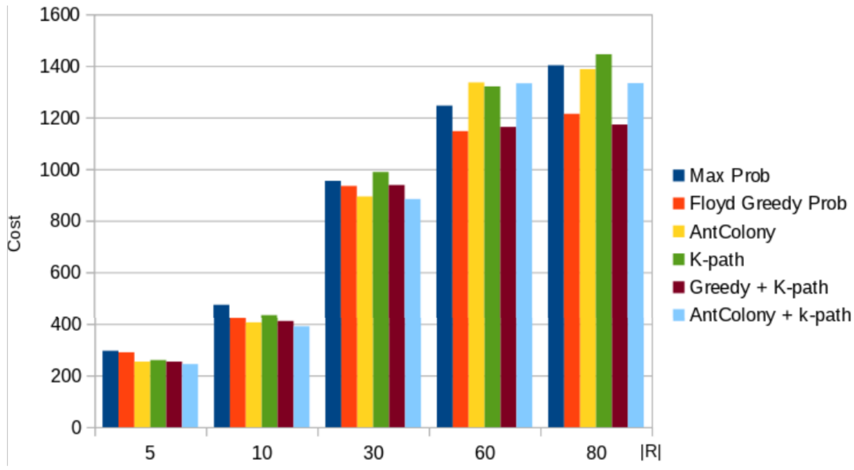


FIGURE 5. Numerical experiments

Below we show examples how the algorithms proposed work. As a primary network PS we consider 5×5 grid, numbered from the top left corner to the left then down. $R = \{(1-10), (5-22), (11-24), (20-22)\}$, $P_0 = 0.7$. The rest of the parameters are the same as in the experiment above. Fig. 6 shows the paths for secondary network edges laying found by various algorithms. For each edge, the path obtained is highlighted with the specified color. For each test, the calculation time was within a few seconds.

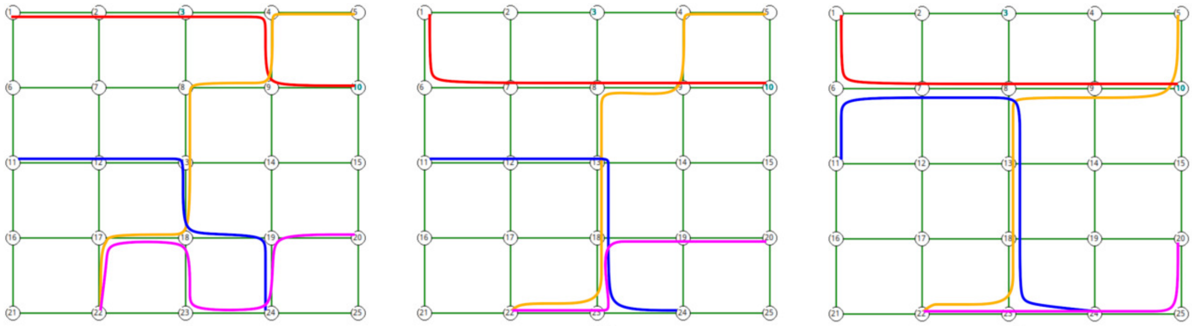
For *MaxProb* algorithm (Fig. 6 a)), $Q(HN) = 186$,
$F(R) = \{(1-2-3-4-9-10), (5-4-9-8-13-18-17-22), (11-12-13-18-19-24), (20-19-24-23-18-17-22)\}$.

For *Greedy+k-path* algorithm (Fig. 6 b)), $Q(HN) = 150$,
$F(R) = \{(1-6-7-8-9-10), (5-4-9-8-13-18-23-22), (11-12-13-18-23-24), (20-19-18-23-22)\}$.

For *AntColony+k-path* algorithm (Fig. 6 c)), $Q(HN) = 128$,
$F(R) = \{(1-6-7-8-9-10), (5-10-9-8-13-18-23-22), (11-6-7-8-13-18-23-24), (20-25-24-23-22)\}$.

7. CONCLUSION

The utility networks optimization problems and their solution methods are investigated. As a model for describing such networks of a hierarchical structure, a two-level hypernet was used, which allows us to take into account a way laying of the upper level network to the lower level network and the interdependence of elements of these networks. Based on it, we formulate the network optimization problems considering their multi-criteria nature.



a) b) c)

FIGURE 6. Paths for secondary network edges laying found by various algorithms

The NP-hardness of the basic problem of a minimum cost two-level hypernet design is proved. We reduce the well-known Steiner problem in graph, which is known to be NP-hard, to this problem. A number of heuristic algorithms based on the k-shortest path method, greedy strategy, ant colony algorithm, methods of the hypernet theory, are proposed.

As numerical experiments have shown, the proposed methods can give us a good approximate solution which is an appropriate compromise between optimization criteria. Future research may be carried out based on investigation of the quality of approximate methods. Another promising area is a more detailed modeling of utility networks with the use of the hypernet apparatus, for example, consideration of failures of elements not only of the primary network.

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